

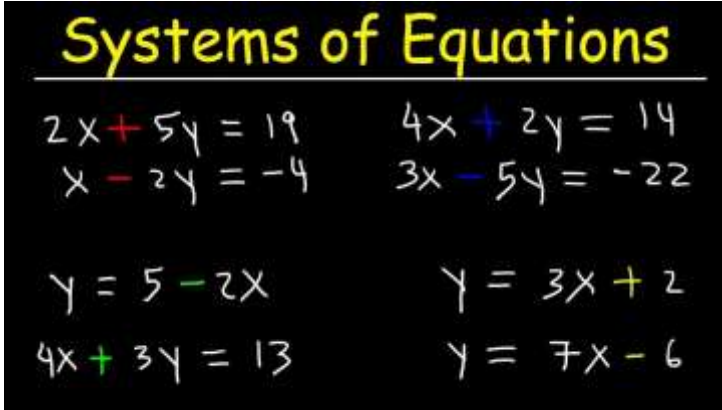


Systems of Equations



Systems of Equations: What are They?

A system of equations is a set of one or more equations involving a number of variables.



Systems of Equations

$$\begin{array}{l} 2x + 5y = 19 \\ x - 2y = -4 \end{array} \quad \begin{array}{l} 4x + 2y = 14 \\ 3x - 5y = -22 \end{array}$$
$$\begin{array}{l} y = 5 - 2x \\ 4x + 3y = 13 \end{array} \quad \begin{array}{l} y = 3x + 2 \\ y = 7x - 6 \end{array}$$

Systems of Equations in Mesopotamia

Mathematicians in Babylon studied both equations and systems of equations. However, they were more interested in the **latter**. They devised and solved linear systems of equations with several unknowns. They called these unknowns *length, width, area, volume...* even if they had nothing to do with geometry.



1	┆	11	< ┆	21	<< ┆	31	<<< ┆	41	<<<< ┆	51	<<<<< ┆
2	┆┆	12	< ┆┆	22	<< ┆┆	32	<<< ┆┆	42	<<<< ┆┆	52	<<<<< ┆┆
3	┆┆┆	13	< ┆┆┆	23	<< ┆┆┆	33	<<< ┆┆┆	43	<<<< ┆┆┆	53	<<<<< ┆┆┆
4	┆┆┆┆	14	< ┆┆┆┆	24	<< ┆┆┆┆	34	<<< ┆┆┆┆	44	<<<< ┆┆┆┆	54	<<<<< ┆┆┆┆
5	┆┆┆┆┆	15	< ┆┆┆┆┆	25	<< ┆┆┆┆┆	35	<<< ┆┆┆┆┆	45	<<<< ┆┆┆┆┆	55	<<<<< ┆┆┆┆┆
6	┆┆┆┆┆┆	16	< ┆┆┆┆┆┆	26	<< ┆┆┆┆┆┆	36	<<< ┆┆┆┆┆┆	46	<<<< ┆┆┆┆┆┆	56	<<<<< ┆┆┆┆┆┆
7	┆┆┆┆┆┆┆	17	< ┆┆┆┆┆┆┆	27	<< ┆┆┆┆┆┆┆	37	<<< ┆┆┆┆┆┆┆	47	<<<< ┆┆┆┆┆┆┆	57	<<<<< ┆┆┆┆┆┆┆
8	┆┆┆┆┆┆┆┆	18	< ┆┆┆┆┆┆┆┆	28	<< ┆┆┆┆┆┆┆┆	38	<<< ┆┆┆┆┆┆┆┆	48	<<<< ┆┆┆┆┆┆┆┆	58	<<<<< ┆┆┆┆┆┆┆┆
9	┆┆┆┆┆┆┆┆┆	19	< ┆┆┆┆┆┆┆┆┆	29	<< ┆┆┆┆┆┆┆┆┆	39	<<< ┆┆┆┆┆┆┆┆┆	49	<<<< ┆┆┆┆┆┆┆┆┆	59	<<<<< ┆┆┆┆┆┆┆┆┆
10	<	20	<<	30	<<<	40	<<<<	50	<<<<<	60	┆

Ancient Near East
empires from 700-300 BCE





A Babylonian Problem

In Babylon, they solved problems using guesswork. They tried different amounts until they got the right solution.

The following problem appeared on Babylonian tablets:

$$\frac{1}{4} \text{ width} + \text{length} = 7 \text{ hands}$$

$$\text{width} + \text{length} = 10 \text{ feet}$$

Breakthrough in China

One of the oldest Chinese mathematics books, the *Nine chapters on the mathematical art* (1st century BCE), includes 246 everyday problems in surveying, engineering, distributions, taxes, etc. The eighth chapter contains systems of equations with up to four unknowns. They used very advanced methods to solve these systems of equations.





Breakthrough in Greece

In 3rd century Alexandria, **Diophantus** devised algebraic problems that could be easily made into systems of equations. He solved them by carefully choosing and naming an unknown that allowed him to create one single equation.

For example, in his book *Arithmetica*, Diophantus devised problems like this one:

Find two numbers that add up to 20 and whose squares add up to 208.

Another Diophantus Problem



Diophantus' book also had more complex problems. To solve them, you need to know about numerical relationships, equations... and use a bit of guesswork.

Here is an example:

A merchant buys jugs of wine, some for 8 drachma and some for 5 drachma. The amount he pays is a square number of drachmas, and the square root of that amount is the same as the number of jugs. How many jugs of wine did he buy for 8 drachma and how many for 5 drachma?

Question

1. How did the Babylonians solve everyday problems with equations?





Answer

Guesswork



Answer

Nine chapters on the mathematical art

Question

How did Diophantus solve systems of equations?





Answer

By carefully choosing and naming an unknown that allowed him to create one single equation.

Question

What did the Babylonians call the unknowns of the systems of equations?





Answer

Length, width, area, volume, etc.

Question

Who wrote the mathematical work *Arithmetica*?





Answer

Diophantus



<https://www.youtube.com/watch?v=-sajLDiqlZE>